The Framed Standard Model (II) - A first Test against Experiment*

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Abstract

Apart from the qualitative features described in [1], the renormalization group equation derived for the rotation of the fermion mass matrices are amenable to quantitative study. The equation depends on a coupling and a fudge factor and, on integration, on 3 integration constants. Its application to data analysis, however, requires the input from experiment of the heaviest generation masses $m_t, m_b, m_\tau, m_{\nu_3}$ all of which are known, except for m_{ν_3}. Together then with the theta-angle in the QCD action, there are in all 7 real unknown parameters. Determining these 7 parameters by fitting to the experimental values of the masses m_c, m_μ, m_e, the CKM elements $|V_{us}|, |V_{ub}|$, and the neutrino oscillation angle $\sin^2\theta_{13}$, one can then calculate and compare with experiment the following 12 other quantities $m_s, m_u/m_d, |V_{ud}|, |V_{cs}|, |V_{tb}|, |V_{cd}|, |V_{cb}|, |V_{ts}|, |V_{td}|, J, \sin^2 2\theta_{12}, \sin^2 2\theta_{23}$, and the results all agree reasonably well with data, often to within the stringent experimental error now achieved. Counting the predictions not yet measured by experiment, this means that 17 independent parameters of the standard model are now replaced by 7 in the FSM.

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In this talk I shall endeavour to quantify the conclusions derived from the framed standard model as described in the previous talk [1], to put actual numbers on what were estimates or inequalities, and to show that indeed the theory is capable of giving a reasonable overall fit to all the data on quark and lepton masses and mixing.

We shall start with a very brief summary of the framed standard model (FSM) [2, 3, 4], pointing out only those of its salient features which we shall refer to. In FSM, frame vectors form part of the geometry of gauge theory, and by promoting these into fields which we call framons, we have built into our system scalar fields which can play the role of the Higgs fields. Moreover they entail a doubling of the gauge symmetry

$$SU(3) \times SU(2) \times U(1) \times \widetilde{SU(3)} \times \widetilde{SU(2)} \times \widetilde{U(1)}.$$

This results in a tree-level mass matrix of rank 1 [5, 6]

$$m = m_T \boldsymbol{\alpha}^\dagger \boldsymbol{\alpha}$$

with no mixing at tree-level. Since the mass matrix is scale-dependent, under renormalization this will lead to non-zero lower generation masses and non-zero mixing [6], according to the following formulae. Denote the state vectors (in generation space) of the t, c, u quarks respectively by $\mathbf{t}$, $\mathbf{c}$, and $\mathbf{u}$ (in the absence of a strong CP phase which does not affect the masses). Then these are obtained by:

$$\begin{aligned}
\mathbf{t} &= \boldsymbol{\alpha}(\mu = m_t); \\
\mathbf{c} &= \mathbf{u} \times \mathbf{t}; \\
\mathbf{u} &= \frac{\boldsymbol{\alpha}(\mu = m_t) \times \boldsymbol{\alpha}(\mu = m_c)}{|\boldsymbol{\alpha}(\mu = m_t) \times \boldsymbol{\alpha}(\mu = m_c)|},
\end{aligned} \tag{1}$$

Using these vectors, the lower generation masses are determined by

$$\begin{aligned}
m_t &= m_U, \\
m_c &= m_U |\boldsymbol{\alpha}(\mu = m_c) \cdot \mathbf{c}|^2, \\
m_u &= m_U |\boldsymbol{\alpha}(\mu = m_u) \cdot \mathbf{u}|^2,
\end{aligned} \tag{2}$$

When we take into account the strong CP phase θ_{CP}, the state vectors become complex and are given by:

$$\begin{aligned}
\tilde{\mathbf{t}} &= \boldsymbol{\alpha}(\mu = m_t), \\
\tilde{\mathbf{c}} &= \cos\omega_U \boldsymbol{\tau}(\mu = m_t) - \sin\omega_U \boldsymbol{\nu}(\mu = m_t) e^{-i\theta_{CP}/2}, \\
\tilde{\mathbf{u}} &= \sin\omega_U \boldsymbol{\tau}(\mu = m_t) + \cos\omega_U \boldsymbol{\nu}(\mu = m_t) e^{-i\theta_{CP}/2},
\end{aligned} \tag{3}$$

The same procedure applies to the down quark triad.

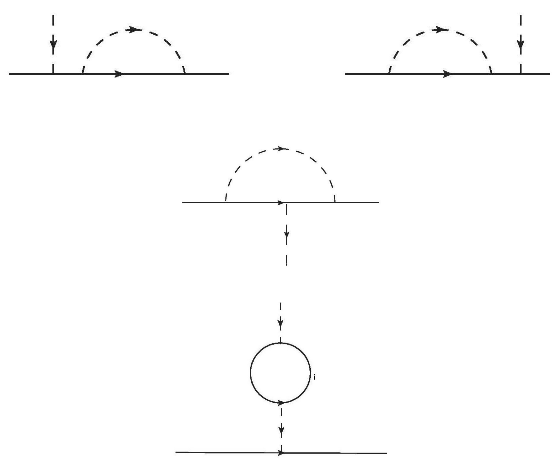
The direction cosines between these two triads then give the CKM mixing matrix:

$$V_{CKM} = \begin{pmatrix} \tilde{\mathbf{u}}\cdot\tilde{\mathbf{d}} & \tilde{\mathbf{u}}\cdot\tilde{\mathbf{s}} & \tilde{\mathbf{u}}\cdot\tilde{\mathbf{b}} \\ \tilde{\mathbf{c}}\cdot\tilde{\mathbf{d}} & \tilde{\mathbf{c}}\cdot\tilde{\mathbf{s}} & \tilde{\mathbf{c}}\cdot\tilde{\mathbf{b}} \\ \tilde{\mathbf{t}}\cdot\tilde{\mathbf{d}} & \tilde{\mathbf{t}}\cdot\tilde{\mathbf{s}} & \tilde{\mathbf{t}}\cdot\tilde{\mathbf{b}} \end{pmatrix}$$

with a complex phase corresponding to the Kobayashi-Maskawa phase giving CP violation. Thus we see explicitly how the QCD θ angle is transformed via rotation into the Kobayashi-Maskawa phase [7].

The case of the leptons is similar, except that we are free not to consider a CP violating phase in the PMNS matrix (as not yet known experimentally).

With these formulae in hand our task is to confront them with actual data, and our main object of interest is the rotating vector $\boldsymbol{\alpha}$. By definition it is a unit vector, and under renormalization it rotates on the unit sphere, tracing out a trajectory say Γ. To study this we compute to 1-loop the relevant Feynman diagrams below which are self-energy diagrams involving the exchange of framons.



Recalling that the framon potential is given by [3, 4]:

$$\begin{aligned} V[\boldsymbol{\alpha}, \boldsymbol{\phi}, \Phi] &= -\mu_W|\boldsymbol{\phi}|^2 + \lambda_W(|\boldsymbol{\phi}|^2)^2 \\ &\quad -\mu_S\sum_{\tilde{a}}|\boldsymbol{\phi}^{\tilde{a}}|^2 + \lambda_S\left(\sum_{\tilde{a}}|\boldsymbol{\phi}^{\tilde{a}}|^2\right)^2 + \kappa_S\sum_{\tilde{a},\tilde{b}}|\boldsymbol{\phi}^{\tilde{a}*}\cdot\boldsymbol{\phi}^{\tilde{b}}|^2 \\ &\quad +\nu_1|\boldsymbol{\phi}|^2\sum_{\tilde{a}}|\boldsymbol{\phi}^{\tilde{a}}|^2 - \nu_2|\boldsymbol{\phi}|^2|\sum_{\tilde{a}}\alpha^{\tilde{a}}\boldsymbol{\phi}^{\tilde{a}}|^2 \end{aligned} \tag{4}$$

we deduce the renormalization equation for $\boldsymbol{\alpha}$.

If we write $\boldsymbol{\alpha}$ in spherical polar coordinates as usual

$$\boldsymbol{\alpha} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix} \tag{5}$$

and introduce the parameter

$$R = \frac{\zeta_W^2 \nu_2}{2\kappa_S \zeta_S^2}, \tag{6}$$

we obtain RGE for the parameters of $\boldsymbol{\alpha}$ as follows:

$$\dot{R} = -\frac{3\rho_S^2}{16\pi^2}\frac{R(1-R)(1+2R)}{D}\left(4 + \frac{R}{2+R} - \frac{3R\cos^2\theta}{2+R}\right) \tag{7}$$

$$\dot{\theta} = -\frac{3\rho_S^2}{32\pi^2}\frac{R\cos\theta\sin\theta}{D}\left(12 - \frac{6R^2}{2+R} - \frac{3k(1-R)(1+2R)}{2+R}\right) \tag{8}$$

and

$$\cos\theta\tan\phi = a \text{ (constant)} \tag{9}$$

where

$$D = R(1+2R) - 3R\cos^2\theta + k(1-R)(1+2R). \tag{10}$$

Here a dot denotes differentiation with respect to $t = \log\mu^2$.

The trajectory Γ traced out by $\boldsymbol{\alpha}$ on the sphere as we vary the scale μ depends on two functions (of scale) which we may call the shape function and the speed function. The shape function is a consequence of symmetry and depends only on one real parameter a (9); so it is simple to deal with, and has been discussed in some detail in [1] (see Figure 1). The speed function, on the other hand, is much less precisely predicted, since first the RGE is only to 1 loop, and secondly it depends on 3 parameters ρ_S and two integration constants. Even more seriously it depends on some unknown and perhaps uncalculable effects represented by the function $k(\mu)$. Clearly one can do little phenomenologically with an unknown function. With some justification we replace it by a constant k.

Before we actually confront our theory with the fermion masses and mixing data, let us see what kind of data we are faced with. There are three points to note.

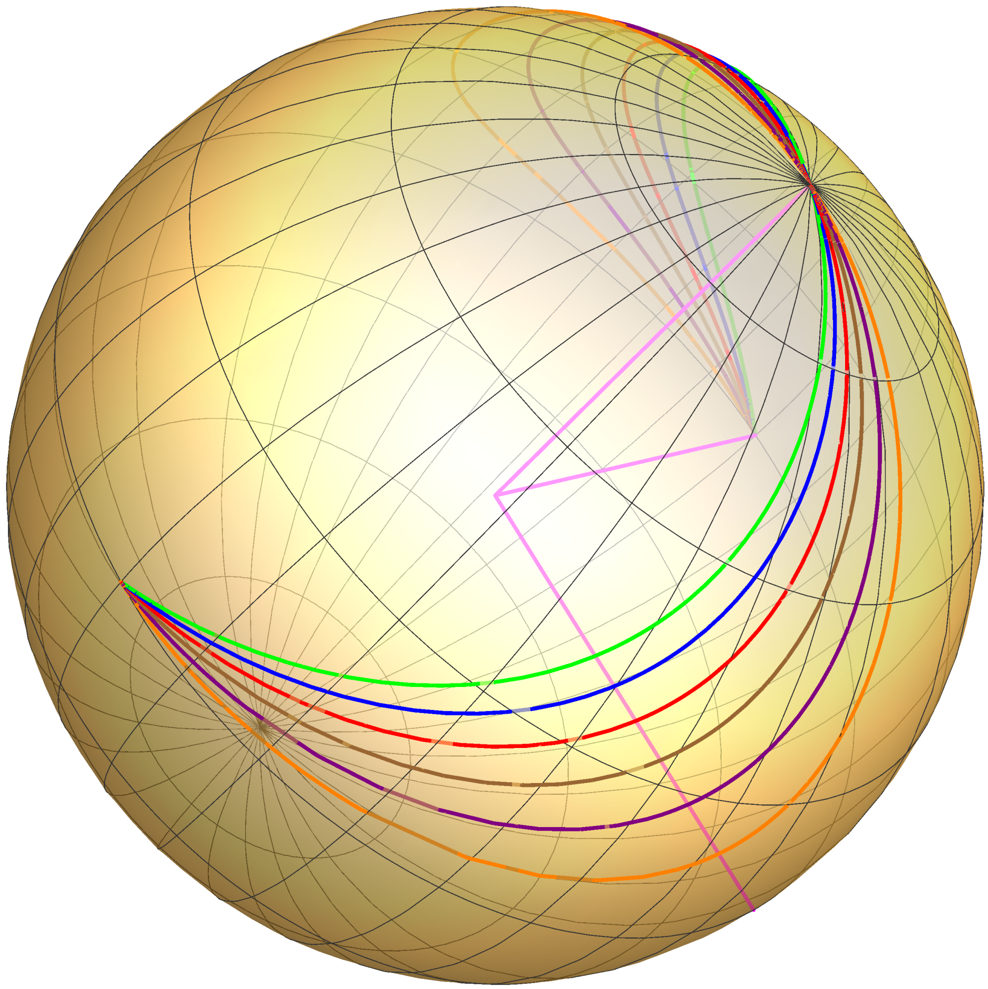


Figure 1: The curve Γ traced out by the vector $\boldsymbol{\alpha}$ on the unit sphere in generation space for various values of the integration constant a, decreasing in magnitude from $a = -0.6$ in green to $a = -0.1$ in orange.

- There is a large amount of data.
- They have vastly different percentage errors.
- The masses range over 13 orders of magnitude.

To be concrete, let me quote the data as given by the PDG [8] in the summer of 2014, when we did the fit to be reported below [4].

The quark masses:

$$\begin{aligned}
m_t &= 173.07 \pm 0.52 \pm 0.72 \text{ GeV} \\
m_c &= 1.275 \pm 0.025 \text{ GeV} \\
m_u &= 2.3^{+0.7}_{-0.5} \text{ MeV (at 2 GeV)} \\
m_b &= 4.18 \pm 0.03 \text{ GeV} \\
m_s &= 0.095 \pm 0.005 \text{ GeV (at 2 GeV)} \\
m_d &= 4.8^{+0.5}_{-0.3} \text{ MeV (at 2 GeV)}
\end{aligned}$$

The lepton masses:

$$\begin{aligned}
m_\tau &= 1776.82 \pm 0.16 \text{ MeV} \\
m_\mu &= 105.6583715 \pm 0.0000035 \text{MeV} \\
m_e &= 0.510998928 \pm 0.000000011 \text{ MeV}
\end{aligned}$$

with the squared mass differences for the neutrinos:

$$\begin{aligned}
(m_{\nu_3}^{\text{ph}})^2 - (m_{\nu_2}^{\text{ph}})^2 &= (2.23^{+0.12}_{-0.08}) \times 10^{-3} \text{ eV}^2 \\
(m_{\nu_2}^{\text{ph}})^2 - (m_{\nu_1}^{\text{ph}})^2 &= (7.5 \pm 0.20) \times 10^{-5} \text{ eV}^2
\end{aligned}$$

The quark CKM matrix:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix}$$

$$\begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0004} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

	Expt (September 2014)	Input value
m_t	$173.07 \pm 0.52 \pm 0.72$ GeV	173.5 GeV
m_b	4.18 ± 0.03 GeV	4.18 GeV
m_τ	1776.82 ± 0.16 MeV	1.777 GeV

Table 1: The heaviest fermion from each type as input

$$|J| = \left(2.96^{+0,20}_{-0.16}\right) \times 10^{-5}$$

The neutrino oscillation angles:

$$\begin{aligned}
\sin^2 2\theta_{13} &= 0.095 \pm 0.010 \\
\sin^2 2\theta_{12} &= 0.857 \pm 0.024 \\
\sin^2 2\theta_{23} &> 0.95
\end{aligned}$$

The last two angles are also known as the solar angle and the atmospheric angle, respectively. I think we could appropriately call the first the Daya Bay angle.

We are now ready to vary the parameters of the theory so as to produce a trajectory Γ for $\boldsymbol{\alpha}$, which will fit best the masses and mixing data via the formulae (2) and (3).

Remembering that all the fermions lie on the same trajectory Γ, we fix the positions for each type (U quark, D quark, charged lepton, neutrino) by inputting the heaviest generation masses, as shown in Table 1.

The neutrino masses are assumed to be generated by some see-saw mechanism, so we put in some assumed value of m_{ν_3} for the Dirac mass of the heaviest neutrino to fit the data. The value affects only the lepton sector.

Next we shall do a parameter count in detail. The theory has 7 adjustable parameters (4),

$$a, \rho_S, k, R_I, \theta_I, m_{\nu_3}, \theta_{CP}. \tag{11}$$

We shall choose experimental data to fix these, using the following criteria for the choice:

- that they are sufficient to determine the 7 parameters adequately,
- that they have been measured in experiment to reasonable accuracy,

	Expt (June 2014)	FSM Calc	Agree to
INPUT			
m_c	1.275 ± 0.025 GeV	1.275 GeV	$< 1\sigma$
m_μ	0.10566 GeV	0.1054 GeV	0.2%
m_e	0.511 MeV	0.513 MeV	0.4%
$\lvert V_{us}\rvert$	0.22534 ± 0.00065	0.22493	$< 1\sigma$
$\lvert V_{ub}\rvert$	$0.00351^{+0.00015}_{-0.00014}$	0.00346	$< 1\sigma$
$\sin^2 2\theta_{13}$	0.095 ± 0.010	0.101	$< 1\sigma$

Table 2: The input experimental values compared with calculated values

- that they are sufficiently sensitive to the values of the parameters,
- that they are strategically placed in $t = \ln \mu^2$ over the interesting range,

and we end up with the following choice:

- the masses m_c, m_μ, m_e
- the elements $|V_{us}|, |V_{ub}|$ of the CKM matrix for quarks
- neutrino oscillation angle $\sin^2 2\theta_{13}$.

Because of the special role played by the Cabibbo angle $|V_{us}|$ with respect to the geodesic curvature of the trajectory Γ, it fixes by itself already two of the parameters above (11).

We now demand that, by varing the 7 parameters (11), the calculations give us back the 6 inputted data within the desired accuracy: either within experimental errors or within half a percent, as shown in Table 2.

Note that the functional form for the trajectory for $\boldsymbol{\alpha}$ having already been prescribed by the RGE (7, 8, 9), it is not at all obvious that the 6 targeted quantities can be so fitted with the given 7 parameters. That it can indeed be done to the accuracy stipulated constitutes already quite a nontrivial test.

This test done, we can proceed to calculate the following 23 quantities of the standard model[1]:

[1]By the standard model here, we mean that in which the now established fact, that neutrinos have masses and oscillate, is incorporated. This means it will have to carry the Dirac masses of the neutrinos also as parameters. Further, we count θ_{CP} also as a parameter of the standard model although it is often arbitrarily put to zero.

- 8 lower generation masses
- the absolute values of all 9 CKM elements
- the Jarlskog invariant J
- 3 neutrino oscillation angles
- $m_{\nu 3}$
- θ_{CP}

Of these, 17 $(= 23 - 6)$ are independent in SM

- 8 lower generation masses
- 4 CKM parameters
- 3 neutrino oscillation angles
- m_{ν_3}
- θ_{CP}

and of which 12 $(= 17 - 5)$ can be compared to experiment (the remaining 5 being not yet measured):

- $m_c, m_s, m_\mu, m_e, m_u/m_d$
- 4 CKM parameters
- 3 neutrino oscillation angles

However, we need to check 18 $(= 23 - 5)$ experimental values to ensure that we have good accuracy, although these are not all independent in the standard model. For example, although the CKM matrix is unitary and has only 4 independent parameters, ensuring that only these 4 fall within error does not imply that the remaining elements are within error too. These 18 quantities we checked are listed in Tables 2 and 3.

We note that of the 12 output quantities shown in Table 3, 6 are within experimental error or else (m_μ, m_e) within 0.5 percent of the accurate measured values, while 2 are within $\sim 1.5\sigma$. Of the remaining 4, 1 (m_s) can only be roughly compared with experiment, because of QCD running, and it does

	Expt (June 2014)	FSM Calc	Agree to
OUTPUT			
m_s	0.095 ± 0.005 GeV (at 2 GeV)	0.169 GeV (at m_s)	QCD running
m_u/m_d	0.38—0.58	0.56	$< 1\sigma$
$\lvert V_{ud}\rvert$	0.97427 ± 0.00015	0.97437	$< 1\sigma$
$\lvert V_{cs}\rvert$	0.97344 ± 0.00016	0.97350	$< 1\sigma$
$\lvert V_{tb}\rvert$	$0.999146^{+0.000021}_{-0.000046}$	0.99907	1.65σ
$\lvert V_{cd}\rvert$	0.22520 ± 0.00065	0.22462	$< 1\sigma$
$\lvert V_{cb}\rvert$	$0.0412^{+0.0011}_{-0.0005}$	0.0429	1.55σ
$\lvert V_{ts}\rvert$	$0.0404^{+0.0011}_{-0.0004}$	0.0413	$< 1\sigma$
$\lvert V_{td}\rvert$	$0.00867^{+0.00029}_{-0.00031}$	0.01223	41 %
$\lvert J\rvert$	$\left(2.96^{+0,20}_{-0.16}\right) \times 10^{-5}$	2.35×10^{-5}	20 %
$\sin^2 2\theta_{12}$	0.857 ± 0.024	0.841	$< 1\sigma$
$\sin^2 2\theta_{23}$	> 0.95	0.89	$> 6\%$

Table 3: The calculated output values using inputs in Table 2

so compare quite reasonably. The other 3: $|V_{td}|, J, \sin^2 2\theta_{23}$, are all outside the stringent experimental errors, but still not outrageously so. Besides, $|V_{td}|$ and J both being small and therefore delicate to reproduce, obtaining them with the right order of magnitude as they are here is already no mean task.

The fit gives in addition the following values for the 5 other standard model parameters which, not being measured, cannot be checked against experiment at present:

$$\theta_{CP} = 1.78, \quad m_u(\mu = m_u) = 0.22 \text{ MeV [or } m_d(\mu = m_d) = 0.39 \text{ MeV]},$$

$$m_{\nu_3} = 29.5 \text{ MeV}, \quad m_{\nu_2} = 16.8 \text{ MeV}, \quad m_{\nu_1} = 1.4 \text{ MeV}.$$

The Figures 2 and 3 show the actual trajectory of $\boldsymbol{\alpha}$ corresponding to the fit above. There are many interesting features, in accordance with the qualitative expectations described in [1]. Here we would like to comment on one particular aspect.

As already mentioned in [1], because the change in sign of the geodesic curvature of Γ, we have, generically as a consequence of symmetry and not only for this fit, that $m_u < m_d$, despite the fact that $m_t \gg m_b, m_c \gg m_s$. Thus we are able to reproduce this crucial empirical fact, without which the

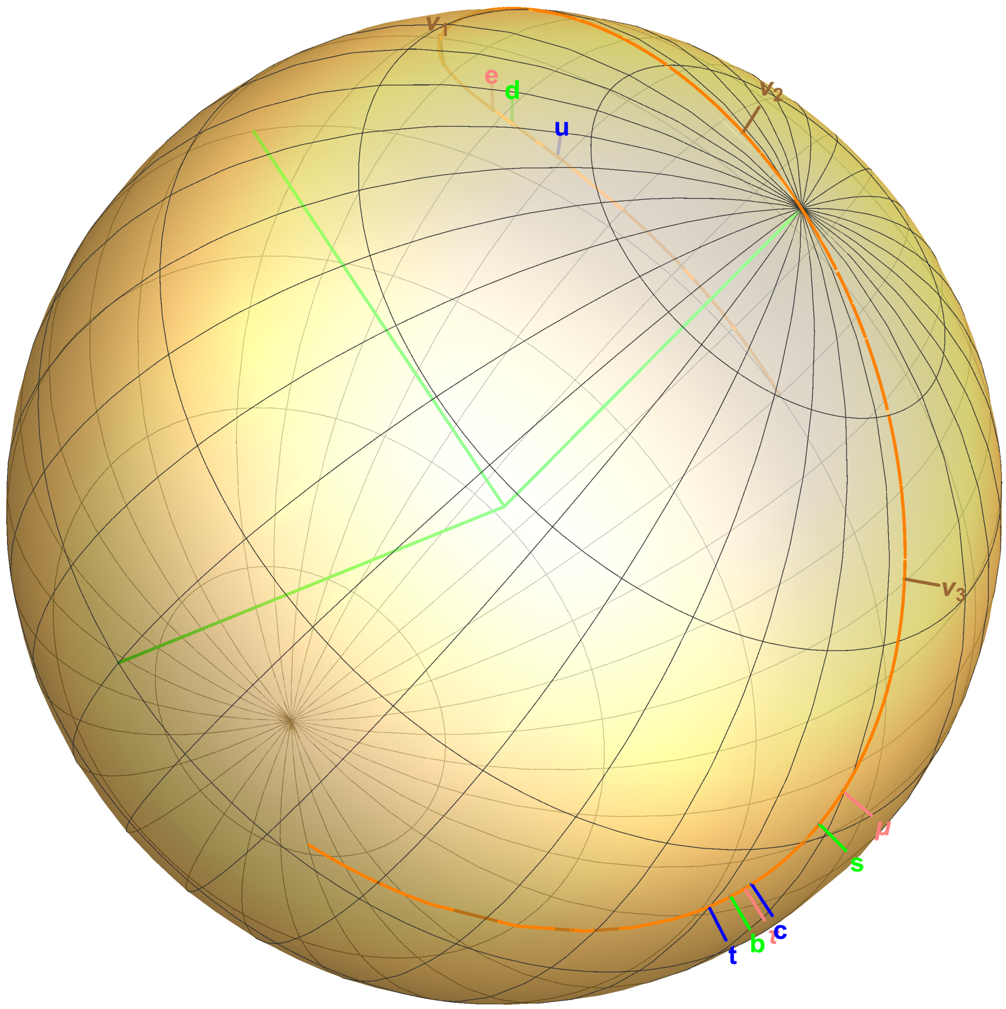


Figure 2: The trajectory for $\boldsymbol{\alpha}$ on the unit sphere in generation space obtained from the parameter values obtained as described, showing the locations on the trajectory where the various quarks and leptons are placed: high scales in front.

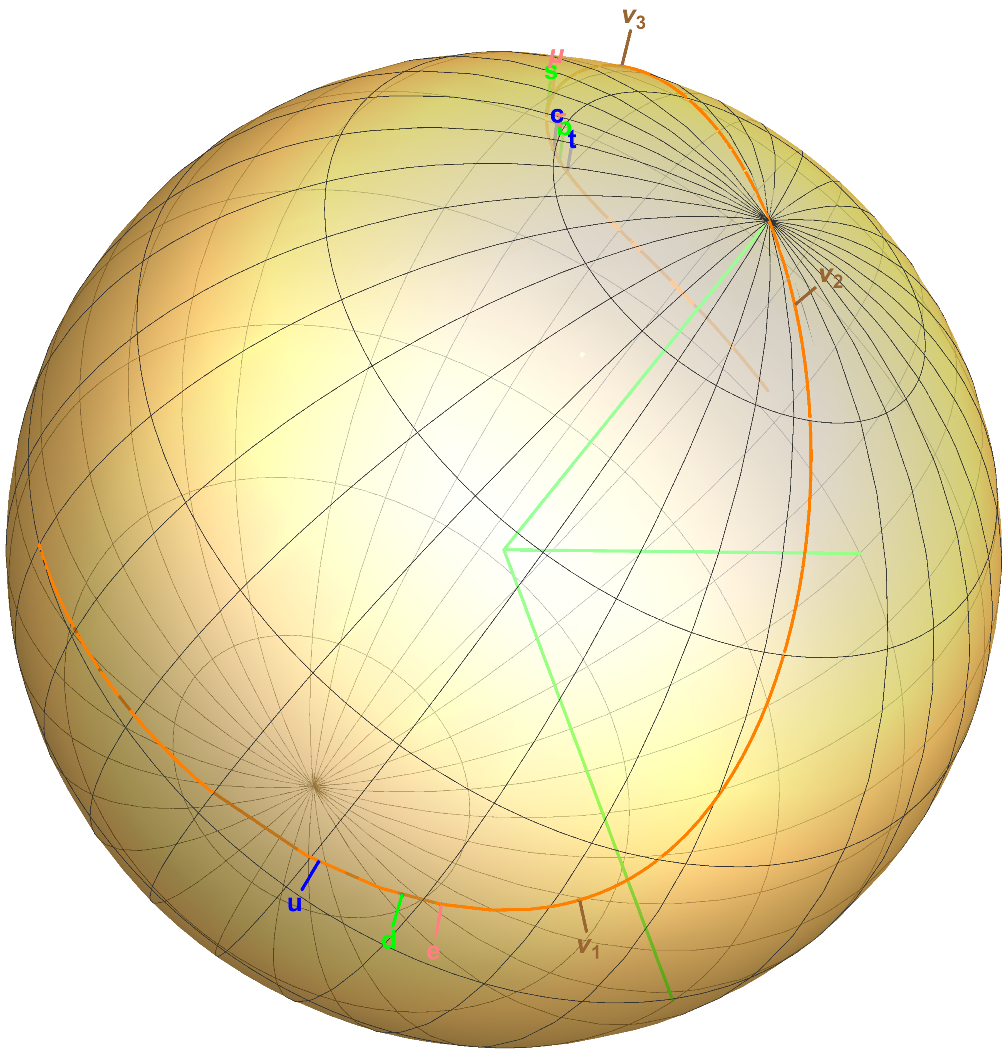


Figure 3: The trajectory for $\boldsymbol{\alpha}$ on the unit sphere in generation space obtained from the parameter values obtained as described, showing the locations on the trajectory where the various quarks and leptons are placed: low scales in front.

proton would be unstable and we ourselves would not exist. To understand this result a bit further, we note that the geodesic curvature changes sign around the scale of order MeV, where masses of the lowest generation quarks occur. Now according to (2) above, the mass of the u and d quarks in FSM are to be given respectively by solution of the equations:

$$|\langle \mathbf{u}|\boldsymbol{\alpha}(\mu)\rangle|^2 = \mu, \quad |\langle \mathbf{d}|\boldsymbol{\alpha}(\mu)\rangle|^2 = \mu, \tag{12}$$

where $\mathbf{u}$ the state vector of u is of course orthogonal to $\mathbf{t}$ and $\mathbf{c}$, the state vectors of t and c. Similarly for the triad $\mathbf{b}, \mathbf{s}, \mathbf{d}$. The masses of u (d) being only of order MeV, this means that one has an approximate solution for $m_u(m_d)$ whenever the vector $\boldsymbol{\alpha}$ crosses the **tc**-plane (**bs**-plane). Given the ordering of the masses of t, b and that, as noted before, $m_c/m_t < m_s/m_b$, the picture is as shown in Figure 4. It is thus clear that in the MeV region where the geodesic curvature has the opposite sign to that in the high scale region, the vector $\boldsymbol{\alpha}$ must cross the **bs**-plane before (i.e. at a higher scale than) the **tc**-plane. In other words, m_d must be larger than m_u, as experiment wants.

It is interesting to note that a scale of a few MeV (at which our geodesic curvature changes sign) occurs also, but for a different reason, in another rotating mass scheme [9] quite similar to ours.

In summary, we can say that

- with 7 adjustable parameters
- can calculate 23 quantities
- of which 18 are measurable
  - 10 within errors
  - 2 within 0.5 %
  - 2 within $\sim 1.5\sigma$
  - 3 within order of magnitude (or better)
  - 1 with QCD running (cannot calculate at present)
- 17 independent in SM
- 12 both measurable and independent
- bonus point: $m_d > m_u$ generically

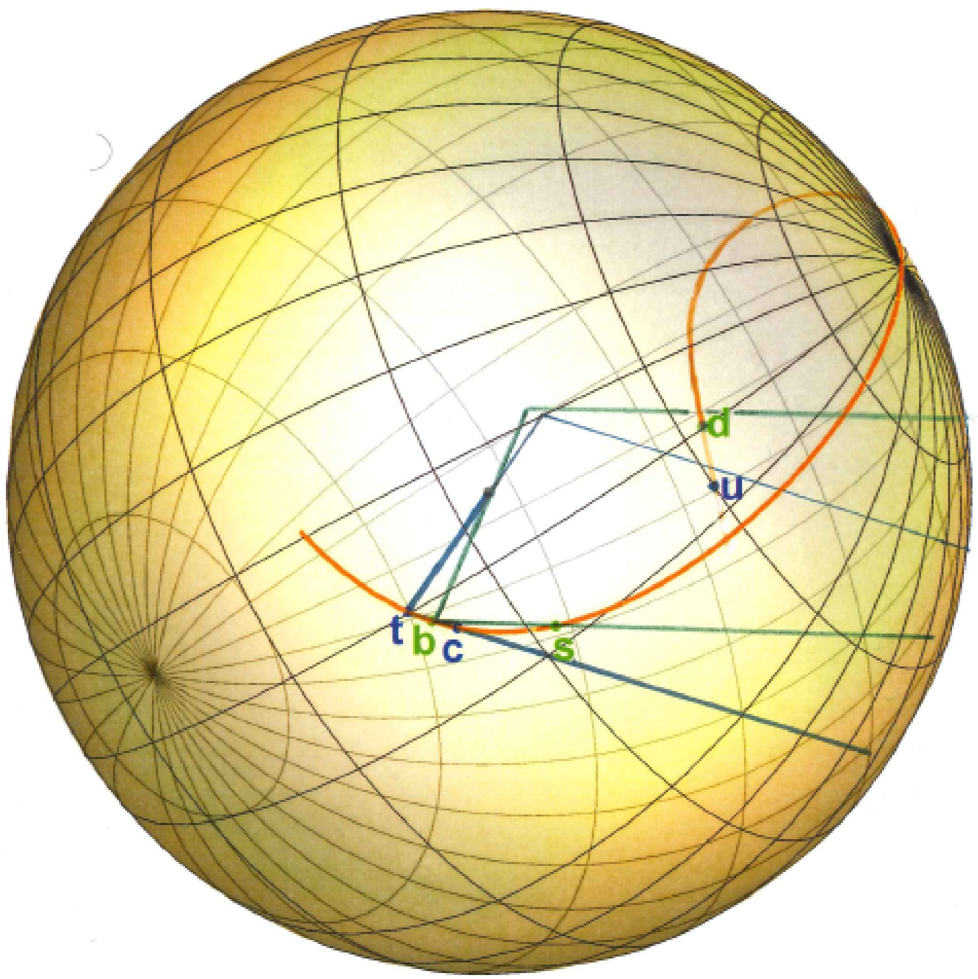


Figure 4: Figure illustrating the reason why $m_u < m_d$ in Table 3.

In this short talk I did not present the framed standard model in full, but only a small part of it: a fit to data. We did not explore all parameter space; our purpose was just to show that it is possible to obtain a decent (or to our biased eyes, a good) fit. This fit fixes for us a number of parameters in the theory, which we shall use to further explore consequences of FSM.

The work reported was done in collaboration with Jose Bordes.

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